5. M. G. Musaev, É. K. Chekalin, and L. V. Chernykh, "Properties of a contracted discharge in an inhomogeneous boundary layer at an electrode in a high-velocity plasma flow," Prikl. Mekh. Tekh. Fiz., No. 3 (1983).
6. V. A. Tishchenko, N. V. Khandurov, and É. K. Chekalin, "Study of ionization processes behind strong shock waves using a conductivity sensor," Zh. Tekh. Fiz., 14, No. 5 (1974).
7. A. V. Bogdans, V. A. Bashilov, V. M. Gribkov, et al., "Study of the regime of transient cathode spots on electrodes in the channel of an MHD generator," in: First Soviet-American Colloquium on MHD Energy Conversion [in Russian], Nauka, Moscow (1974).

OPTIMIZATION OF THE INTERNAL SOURCE IN THE PROBLEM OF MHD FLOW AROUND A SPHERE

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Studies on the effect of electromagnetic body forces on the hydrodynamic pattern of the flow around bodies propelled by internal sources of electromagnetic fields have become of interest in connection with the design of magnetohydrodynamic propellers for submarines and surface vessels (see, e.g., [1, 2] and their bibliographies). Such studies for the case of a sphere as an example were started in [3-5] for fixed sources, which were chosen from qualitative considerations and are not optimum.

Clearly, the distributions of the electric and magnetic potentials at the surface of the sphere should be optimum, ensuring that the electrical energy consumption for propulsion at a given speed be minimum. Our goal here is to formulate the complete variational problem for determining the optimum potentials, construct the solutions of some simplified (variational) problems, and analyze them.

1. We consider a sphere of radius a with an internal source of fields, which was described in [3]. Electromagnetic fields in a liquid are characterized by the scalar potentials

$$\mathbf{E} = -\nabla[\varphi(r, \theta) \sin m\alpha], \quad \mathbf{B} = -\nabla[\chi(r, \theta) \cos m\alpha]. \tag{1.1}$$

The velocity field is assumed to be axisymmetric,

$$\mathbf{v} = \frac{1}{r \sin \theta}\left(-\frac{1}{r}\frac{\partial \psi}{\partial \theta}\mathbf{e}_r + \frac{\partial \psi}{\partial r}\mathbf{e}_\theta\right), \tag{1.2}$$

and is described by the stream function $\psi(r, \theta)$ (the sense of the axisymmetry assumption and some considerations concerning its applicability are given in [3]). The functions $\phi(r, \theta)$, $\chi(r, \theta)$, $\psi(r, \theta)$, and $w(r, \theta)$ [vorticity $\operatorname{curl} \mathbf{v} = w(r, \theta)\mathbf{e}_\alpha$] are determined from the problem

$$L\varphi = \frac{m\chi}{r \sin \theta} w, \quad L\chi = 0 \tag{1.3}$$

$$\left(L = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} - \frac{m^2}{r^2 \sin^2\theta}\right);$$

$$-\frac{1}{2r}\left[\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\frac{w}{r\sin\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}\frac{w}{r\sin\theta}\right] + \frac{1}{\mathrm{Re}}\frac{1}{r\sin\theta}E^2(r\sin\theta w) + N\langle \operatorname{curl}_\alpha \mathbf{f}\rangle = 0; \tag{1.4}$$

$$E^2\psi - rw\sin\theta = 0 \quad \left(E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right); \tag{1.5}$$

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$$\chi(1,\theta)=\chi_0(\theta),\ \varphi(1,\theta)=\varphi_0(\theta),\ \psi(1,\theta)=0,\ \frac{\partial\psi}{\partial r}(1,\theta)=0, \tag{1.6}$$
$$\chi|_{r=\infty}=\varphi|_{r=\infty}=0,\ \psi|_{r\to\infty}=\frac{1}{2}r^2\sin^2\theta,\ w|_{r=\infty}=0.$$

The undetermined functions $\chi_0(\theta)$ and $\phi_0(\theta)$ in the boundary conditions (1.6) should be found by solving the variational problem of minimizing the functional representing the electric power consumption $f_0u_0a^3\iiint(\mathbf{j}\cdot\mathbf{E})\,dv$ (integral over the space outside the sphere). The problem is simplified if $Q=\iiint(\mathbf{j}\cdot\mathbf{E})\,dv$ is written as an integral over the surface of the sphere. In the general case (when $\mathbf{j}$ is expressed in terms of curl $\mathbf{H}$) this transition is made by using the Poynting vector; a corresponding representation is also possible in the inductionless approximation under consideration since $\mathbf{j}\cdot\mathbf{E}=-\mathrm{div}(\phi\mathbf{j})=-\mathrm{div}\,\phi[\mathbf{E}+\mathbf{v}\times\mathbf{B}]$ because div $\mathbf{j}=0$. Hence

$$Q=\int_0^\pi\varphi(1,\theta)\left[-\frac{\partial\varphi(1,\theta)}{\partial r}+\frac{m\chi(1,\theta)}{\sin\theta}v_\theta(1,\theta)\right]\sin\theta\,d\theta. \tag{1.7}$$

(The conditions of sticking on the sphere have not yet been taken into account so that this form of the functional Q could also be used for an ideal liquid.)

The sphere under consideration is self-propelled. This means that zero electromagnetic tractive force acts on the sphere. This condition can be written as

$$8N\int_0^\pi\int_1^\infty\{\langle f_r(r,\theta)\rangle\cos\theta-\langle f_\theta(r,\theta)\rangle\sin\theta\}r^2\sin\theta\,d\theta\,dr+ \tag{1.8}$$
$$+\int_0^\pi\left\{\frac{4}{\mathrm{Re}}\left[w(1,\theta)-\frac{\partial w(1,\theta)}{\partial r}\right]-4N\langle f_\theta(1,\theta)\rangle\right\}\sin^2\theta\,d\theta=0.$$

The forces $\langle f_r\rangle$ and $\langle f_\theta\rangle$ (the angular brackets denote averaging over the angle α) are expressed in terms of the functions ϕ, χ, ψ and their derivatives from formulas in [6].

The variational problem under study thus is a problem for an angular extremum and consists of choosing the functions ϕ, χ, ψ, w that would ensure the minimum of the functional Q (1.7) with the auxiliary conditions (1.3)-(1.6), the condition of self-propulsion (1.7), and a condition limiting the scale of the magnetic field. It is most desirable from the physical standpoint to formulate the last condition as the requirement that the maximum value of the dimensionless magnetic induction at the surface of the sphere be equal to 1,

$$|B|_{\max}|_{r=1}=1 \tag{1.9a}$$

(the B_0 used when the scale is made dimensionless is the maximum magnetic induction at the surface). The simple limitation

$$(B_\alpha)_{\max}|_{r=1}=\left(\frac{m\chi(1,\theta)}{\sin\theta}\right)_{\max}=1 \tag{1.9b}$$

(B_0 is the maximum value of B_α) is meaningful. Such a limitation cannot be easily implemented directly in a variational problem. The imposition of a limitation on the functional of $\chi(1,\theta)$ is more suited to the nature of the variational problem. Here we consider limitations of the type

$$\int_0^\pi h_1(\theta)\chi(1,\theta)\,d\theta=\mathrm{const} \tag{1.10a}$$

or

$$\int_0^\pi h_2(\theta)\chi^2(1,\theta)\,d\theta=\mathrm{const} \tag{1.10b}$$

with given functions $h_1(\theta)$ and $h_2(\theta)$, the constants in (1.10a) and (1.10b) being chosen so that the solutions obtained would ensure satisfaction of (1.9b).

2. Using indeterminate Lagrangian multipliers we reduce the formulated variational problem to a nonlinear system of eight equations in partial derivatives (for the variables ϕ, χ, ψ, w and four Lagrangian multipliers) with the corresponding nonlinear boundary conditions [6]. The problem is very difficult to solve.

As the first step we consider the simplest problems, which arise if the velocity field around the sphere is replaced by a more appropriate approximation, e.g., the velocity field of potential flow with the stream function

$$\psi = (1/2)(r^2 - 1/r)\sin^2\theta, \tag{2.1}$$

and the hydrodynamic variables are eliminated from the list of functions sought. The first of two simplified problems (problem A) is formulated as follows: given the velocity field (determined from (2.1)), find the optimum distributions $\chi_0(\theta)$ and $\phi_0(\theta)$ for which the functional

$$F = -2\pi\int_0^\pi\int_1^\infty [\langle f_r(r,\theta)\rangle\cos\theta - \langle f_\theta(r,\theta)\rangle\sin\theta]\, r^2\sin\theta\, dr\, d\theta \tag{2.2}$$

(dimensionless tractive force) takes on a fixed value and the electric power consumption Q from (1.7) becomes a minimum.

The question of how electromagnetic body forces affect the velocity field and the hydrodynamic drag cannot be raised in this formulation and so instead of the self-propulsion requirement (1.8) the condition that a given quantity F be the parameter of the variational problem is used. As a result this change the optimum internal source, constructed by solving the simplified problem, gives the minimum electric power consumption for generating a given tractice force and not for ensuring a given speed of propulsion. Since the required tractive force is not known in advance (since the hydrodynamic drag depends entirely on the operation of the source), the problem under consideration does not make sense at first glance. This conclusion is correct if the structure of the optimum source depends on F and specific optimum functions $\chi_0(\theta)$ and $\phi_0(\theta)$ correspond to each value of F. If the simplified variational problem leads to a universal optimum source that is independent of F, the solution of this problem is of unquestionable interest and an internal source so constructed can be used to minimize the power consumption for ensuring the desired speed. We emphasize that such a "universal" solution still does not give a solution for the complete variational problem formulated in Sec. 1, since the possibility of minimizing Q by deliberately controlling the hydrodynamic drag is not envisaged for the simplified problems. One more comment about the formulation of the simplified variational problems: Inasmuch as Q as well as F generally do not depend on the velocity field, a question naturally arises as to the validity of using (2.1) when calculating the integrated quantities. We give some arguments in support of this assumption and reinforce them below with numerical results. Since in the self-propulsion regime the flow around the sphere, as shown in previous studies, is flow without separation, the flow vorticity affecting the electric field distribution in the liquid (see (1.3)) is concentrated mainly inside the boundary layer [3]. Since at high Re values the boundary layer may be substantially thinner than the region occupied by the electromagnetic body forces, the indicated integrated quantities are determined primarily by the electromagnetic fields outside the boundary layer, where the vorticity of the flow and its effect on the field distribution can be assumed to be zero and its effect on the velocity field is that which stems from the assumption (2.2). The potential distributions satisfy the homogeneous equations

$$L\varphi = 0, L\chi = 0. \tag{2.3}$$

Elsewhere [6] we showed that for a given variational problem the potentials on the surface of a sphere are related by

$$\partial\varphi/\partial r|_{r=1} = k_1\chi(1,\theta), k_1 = \text{const}. \tag{2.4}$$

Since the derivative $r\partial\phi/\partial r$ satisfies Eq. (2.3), then from (2.4) we have

$$r\partial\varphi(r,\theta)/\partial r = k_1\chi(r,\theta), \tag{2.5}$$

and, therefore, in the optimized system only one of the two sought distributions $\phi_0(\theta)$ and $\chi_0(\theta)$ remains independent (e.g., assignment of $\phi_0(\theta)$ to within a constant factor from (2.5) determines $\chi_0(\theta)$ and conversely). We also showed that if a solution of problem A is to exist, the limitation on the scale of the field B must have one of the forms of (1.10) and (1.9a) or (1.9b) cannot be used directly. The form of the optimum function $\chi_0(\theta)$ depends on h_1 and h_2 from (1.10), and a specific optimum distribution corresponds to each value of F, where by problem A does not lead to a universal optimum function $\chi_0(\theta)$.

3. For the transition to the formulation of a new problem we note that condition (2.4) does not depend on (1.10) and is the result necessary for minimizing Q. In fact (2.4) implies that in the optimum system the radial components of the fields $\mathbf{E}$ and $[\mathbf{v} \times \mathbf{B}]$ in Ohm's law should be proportional to each other over the entire surface of the sphere, i.e.,

$$\{E_r\}|_{r=1} = -k\{[\mathbf{v} \times \mathbf{B}]_r\}|_{r=1}. \tag{3.1}$$

An analog of condition (3.1) was obtained in [7] for an infinitely wide plate.

We formulate a modified variational problem (problem B) as follows: for a given value of k find the optimum distribution $\chi_0(\theta)$ (or $\phi_0(\theta)$) that would ensure maximum efficiency $\eta = F/Q$. This problem is important because the sought function $\chi_0(\theta)$ is universal and independent of k. Moreover, the normalization condition (1.9b) is satisfied directly here, without the auxiliary limitations (1.10) that are necessary for problem A to have a solution. As a result, the optimum distribution $\chi_0(\theta)$ obtained ensures a high η for all F except those found by solving problem A.

To solve the problem we note that the functionals F and Q under consideration are expressed in terms of the potential $\phi(r, \theta)$ when (2.5) is used. In this case

$$Q = \pi\left(\frac{1}{k} - 1\right)\int_0^\pi \varphi(1, \theta)\,\frac{\partial\varphi(1, \theta)}{\partial r}\sin\theta d\theta, \tag{3.2}$$

and in the expression for F we can also isolate a part that represents a surface integral. Indeed, since the first term in $f_z = [\mathbf{E} \times \mathbf{B}]_z + [(\mathbf{v} \times \mathbf{B}) \times \mathbf{B}]_z$ is put into the divergence form $[\mathbf{E} \times \mathbf{B}]_z = \operatorname{div}\,[\mathbf{e}_z \times \varphi\ \sin m\alpha \mathbf{B}]$, the expression $F = -\int\int\int f_z dv$ reduces to

$$F = -\pi m\int_0^\pi \varphi(1, \theta)\,\chi(1, \theta)\sin\theta d\theta - 2\pi\int_0^\pi\int_1^\infty \langle[(\mathbf{v}\times\mathbf{B})\times\mathbf{B}]_z\rangle r^2\sin\theta dr\,d\theta$$

or, when (2.5) is taken into account, to the final form

$$F = -\frac{2\pi}{3k}\int_0^\pi \varphi(1, \theta)\frac{\partial\varphi(1, \theta)}{\partial r}\sin\theta d\theta - \pi\left(\frac{2}{3mk}\right)^2\int_0^\pi\int_1^\infty\{\ldots\}\,dr\,d\theta, \tag{3.3}$$

where $\{\ldots\} = \left\{\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) r\frac{\partial^2\varphi}{\partial r\,\partial\theta}\left(\frac{\partial\psi}{\partial r}\cos\theta + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\sin\theta\right) + \left(\frac{m}{\sin\theta}\frac{\partial\varphi}{\partial r}\right)^2\left(\frac{\partial\psi}{\partial\theta}\cos\theta + r\frac{\partial\psi}{\partial r}\sin\theta\right) + \frac{\partial\psi}{\partial\theta}\left(\frac{\partial^2\varphi}{\partial r\,\partial\theta}\right)^2\cos\theta + r\frac{\partial\psi}{\partial r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)\right]^2\sin\theta\right\}$. Then for a given k the relation under study

$$\eta = \frac{F}{Q} = \frac{\frac{2}{3}k + \beta}{k(k-1)}, \quad \beta = \left(\frac{2}{3m}\right)^2\frac{\int_0^\pi\int_1^\infty\{\ldots\}\,dr\,d\theta}{\int_0^\pi \varphi(1, \theta)\frac{\partial\varphi(1, \theta)}{\partial r}\sin\theta d\theta} \tag{3.4}$$

depends on coefficient β, which is a functional $\beta[\phi_0(\theta)]$. We see that the maximum value of η for all k is reached with the same function $\phi_0(\theta)$ that ensures a maximum of β.

Let us assume that such a function $\phi_0(\theta)$ does exist. Like $\chi_0(\theta)$, which is obtained from $\phi_0(\theta)$ by (2.5), it is determined to within an arbitrary multiplier. By an appropriate choice of the normalizing multiplier, therefore, condition (1.9b) can be satisfied without invoking limitations of the type (1.10). We also note that the optimum distribution of $\chi_0(\theta)$ does not depend on k.

TABLE 1

m	β_{max}	m	β_{max}
2	−0,939	20	−0,680
4	−0,770	30	−0,675
10	−0,697		

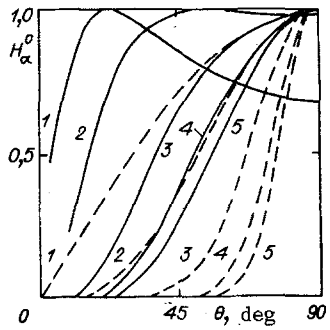

Fig. 1

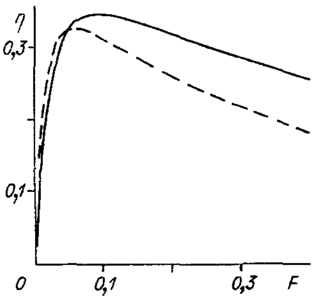

Fig. 2

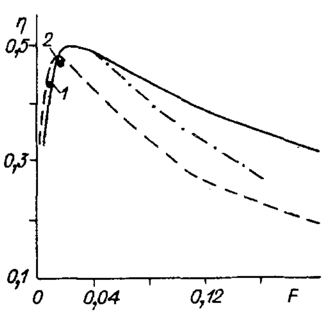

Fig. 3

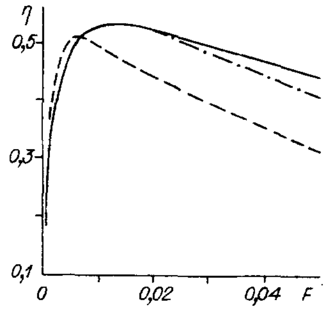

Fig. 4

To find the sought distributions we use solutions of Eqs. (2.3) in the form of expansions in associated Legendre functions:

$$\chi(r,\theta)=\sum_{l=m}^{\infty}\frac{A_l}{r^{l+1}}P_l^m(\cos\theta),\quad \varphi(r,\theta)=-\frac{3km}{2}\sum_{l=m}^{\infty}\frac{A_l}{l+1}\frac{1}{r^{l+1}}P_l^m(\cos\theta). \tag{3.5}$$

The search for an extremum of β reduces to a search for the optimum set of coefficients A_ℓ ($\ell = m, m + 1, \ldots$), which is done numerically. For all the values of the number of pole pairs it proved sufficient to keep 30-40 terms of the series and a further increase in this number did not appreciably change the results.

Figure 1 shows the curves of $B_\alpha{}^0(\theta) = m\chi_0(\theta)$ for $m = 2, 4, 10, 20, 30$ (curves 1-5), characterizing the optimum distribution of $B_\alpha(1, \theta, \alpha) = [m\chi(1, \theta)/\sin\theta] \sin m\alpha$ over the surface of the sphere. The maximum values of β obtained for optimum $\phi_0(\theta)$ are given in Table 1.

Naturally, these values satisfy the condition $\beta_{max} < -2.3$, stemming from the consideration of the limiting value $k = 1$, for which the electric power consumption is zero, as is seen from (3.2). Of course, the electromagnetic tractive force and the efficiency (3.2) cannot be positive. Hence the limitation on β_{max}. The quantities F and η pass through zero when $k_0 > 1$ and so a propulsion occurs and electromagnetic braking corresponds to $1 < k < k_0$. Since the latter is not of interest, the values of k_0 are not calculated here (an analogous situation for a flat plate was investigated in greater detail).

TABLE 2

l	A_l	l	A_l	l	A_l
20	$4{,}021\cdot10^{-2}$	30	$2{,}496\cdot10^{-3}$	36	$7{,}313\cdot10^{-4}$
22	$1{,}827\cdot10^{-2}$	32	$1{,}633\cdot10^{-3}$	38	$4{,}982\cdot10^{-4}$
24	$1{,}032\cdot10^{-2}$	34	$1{,}085\cdot10^{-3}$	40	$3{,}432\cdot10^{-4}$
26	$6{,}233\cdot10^{-3}$				
28	$3{,}893\cdot10^{-3}$				

TABLE 3

k	c_p	c_f	c_d	F	η	q	N_*
1,066	0,018	0,0938	0,112	0,0090	0,432	0,541	9,8
1,161	0.063	0,0862	0,149	0,0174	0,466	0,667	6,75

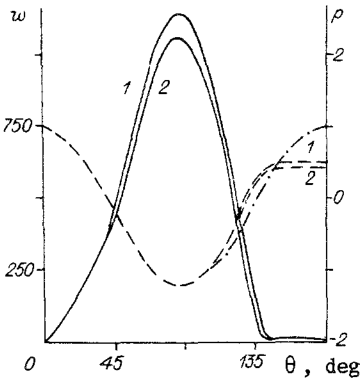

Fig. 5

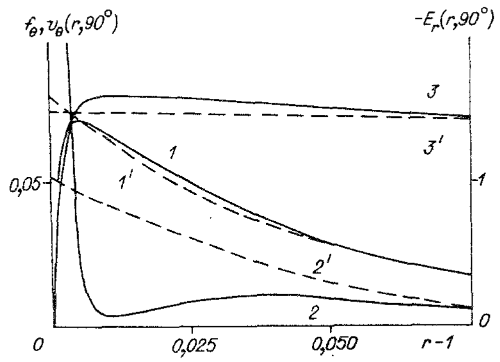

Fig. 6

The $B_\alpha{}^0(\theta)$ profiles are symmetric about the angle $\theta = 90°$ and so they are given between the limits $0 \leq \theta \leq 90°$. The dashed lines here represent $B_\alpha{}^0$ for the multipole $A_{\ell=m} \neq 0$, $A_{\ell>m} = 0$. The optimum profiles are "filled" substantially more than are those of the multipole. We note that $B_\alpha{}^0$ then reaches maximum at $\theta = 90°$ for $m \geq 10$ while for $m = 2$ and 4 the profiles have two maxima located symmetrically about $\theta = 90°$. This qualitative difference is clearly due to the following characteristic features. At large $m \gg 1$ the electromagnetic fields decrease rapidly with distance from the surface of the sphere and are concentrated in a thin surface layer with a thickness of the order of $1/m$, where the velocity field varies little with respect to the radius. At small $m \sim 1$ the electromagnetic fields occupy a region of space outside the sphere with a characteristic dimension of the order of 1, where the distribution $v(r, \theta)$ varies substantially with respect to the angle θ and with respect to the radius. Naturally, qualitatively different optimum distributions of the fields **E**, **B** correspond to the limiting cases.

The β_{max} found (see Table 1) in fact gives the solution of variational problem B in the form $\eta(k)$ (3.4), where F is defined as a function of k by expression (3.3). The calculated results in Figs. 2-4 are given as $\eta(F)$ curves, obtained by eliminating k from the functions $\eta(k)$ and $F(k)$ (solid lines) for m = 10, 20, and 30. The corresponding values of k for each point of the curves are easily reconstructed from (3.4) for η with allowance for β_{max} (see Table 1). We see that the functions $\eta(F)$ are nonmonotonic as in the case of a plate of finite width [7]; for each m there exists a maximum value η_{max}, which is attained at a certain tractive force F. Corresponding to each $\eta < \eta_{max}$ are two values of F, the larger of which is of practical interest. In Figs. 2-4 the $\eta(F)$ curves for the multipole are shown by dashed lines for comparison. The maximum $\eta(F)$ for the optimum distribution are only slightly larger than the corresponding values for the multipole, while the gain in η is substantial for large F. In Figs. 3 and 4 the dash-and-dot lines represent the $\eta(F)$ dependences obtained by solving problem A, using condition (1.10b) with $h_2(\theta) = \sin(\theta)$. These data illustrate the statement that problem A leads to lower values of η than do the results of optimization on formulation B.

We must point out that in all the cases discussed η_{max} are less than (or only slightly exceed) 0.5 for large m when the electromagnetic fields are concentrated in a thin surface layer, where the velocity field depends weakly on the radius (the boundary larger is not taken into account). The situation is very close to that of a flat plate [7] and, it would seem, it should ensure values of η_{max} close to 1. This does not happen for a sphere since the local velocity at the location of the main body forces is 1.5 times the velocity of the sphere and the propulsive efficiency $\approx 1/1.5$. As a result, when m increases η_{max} approaches 2/3 and not 1, as for a flat plate.

4. We give some initial results of numerical studies of MHD flow around a self-propelled sphere, equipped with the optimum source found. These results support the validity of the assumptions that were the basis for the formulation of simplified variational problems and also demonstrate the possibility of a further reduction of the energy expenditure for propulsion of the sphere in comparison with that obtained earlier [5].

As in [5], the calculations were carried out for Re = 10^5 for m = 20 pole pairs. The optimum source is characterized by the function $\chi_0(\theta)$, which is specified by its own expansion coefficients (Table 2):

$$\chi_0(\theta) = \sum_{l=m}^{m+40} A_l P_l^m(\cos\theta),\ P_l^m = \sum_{s=0}^{[(l-m)/2]} (-1)^s \frac{(2l-2s-1)!!}{2^s s!\,(l-2s-m)!} \cos^{l-2s-m}\theta\cdot\sin^m\theta.$$

The distribution $B_\alpha{}^0$ corresponding to this function is shown in Fig. 1. The electric potential at the surface of the sphere is given by a function, which on the basis of (3.5) has the form

$$\varphi_0(\theta) = -\frac{3m}{2} k \sum_{l=m}^{m+40} \frac{A_l}{l+1} P_l^m(\cos\theta)$$

and depends on the load parameter k.

For given Re and m the flow without separation around the sphere has a complex multiscale structure (with the characteristic scale of the problem approximately equal to 1, as determined by the geometry, the boundary layer has a thickness $\sim Re^{-1/2}$ and the region of the force field is characterized by a thickness $\sim m^{-1}$, with $Re^{-1/2} \ll m^{-1} \ll 1$). Accordingly, the calculations were carried out on a family of embedded nets in order to eliminate errors [4]. This requires much computer time and because of this calculations have been made thus far only for two values of k (1.066 and 1.161), the first of which was chosen quite at random while the second corresponds to the highest point on the $\eta(F)$ curve (see Fig. 4) with η = 0.505 for the optimum system.

Figure 5 shows the distributions of the vorticity (solid lines) and pressure (dashed lines) over the surface of the sphere (curves 1, 2 for k = 1.066, 1.161). We see that the distribution of w and p are close for these values of k. The vorticity w is greater than zero over the entire surface, i.e., the flow around the sphere occurs without separation. In the stern region, however, w is less than 1, while $k_{max} \sim 10^3$ and this made it more complicated to obtain a numerical solution. The distribution of p is close to the pressure distribution for nonviscous flow around the sphere (dash-and-dot line), which ensures that the pressure drag coefficient c_p. A "pleateau," where the pressure is roughly constant, exists in the stern region. Table 3 shows the resultant integrated parameters, which characterize the self-propulsion regime.

The values of F and η obtained for each k have been transferred from Table 3 to Fig. 3 (points 1 and 2). These points lie close to the $\eta(F)$ curve obtained as the solution of variational problem B.

In Fig. 6, lines 1, 1'-3, and 3', respectively, give the distribution of $E_r(r, \theta)$ (taken with a minus sign), f_θ, and v_θ (the dashed lines correspond to the nonviscous state). We see that the distributions of $E_r(r, \theta = 90°)$ and $v_\theta(r, \theta = 90°)$ are close to those of nonviscous flow everywhere, except for the boundary-layer region. The distributions of f_θ (r,

$\theta = 90°$) differ more from each other because of the large changes in v_θ, **E**, and **B** in the region under consideration and also because the terms in the expression $f_\theta = \{[\mathbf{E} \times \mathbf{B}] + [(\mathbf{v} \times \mathbf{B}) \times \mathbf{B}]\}_\theta$ have different signs.

The analysis implies that the use of approximation (2.1) to estimate the integrated energy quantities is allowable. Since c_d for self-propulsion depends rather strongly on k, however, it would be desirable to obtain the solution of the complete variational problem without the simplifying assumption (2.1).

The penultimate column of Table 3 contains the parameter q [5], which is the ratio of the electric power consumption and the mechanical power during towing with the same velocity and describes the efficiency of the MHD method of propelling a body. The value q = 0.54 obtained for k = 1.066 shows that the energy consumption for propulsion of a sphere with a given MHD source is only 54% of that for towing and demonstrates that it is possible in principle further to reduce the energy consumption for propulsion of the sphere in comparison with [5] (q = 0.58 in [5]), thus confirming that further research is promising.

LITERATURE CITED

1. Yu. M. Gilerovich and N. N. Smirnova, "Superconducting devices with direct energy conversion," Sudostroenie Rubezhom, No. 5 (1987).
2. A. Iwata, E. Tada, and Y. Saji, "Experimental and theoretical study of superconducting electromagnetic ship propulsion," Fifth Ship Propeller Symposium, Drunen, Netherlands, May (1983).
3. V. I. Shatrov and V. I. Yakovlev, "The hydrodynamic drag of a sphere containing a conduction-type source of electromagnetic fields," Prikl. Mekh. Tekh. Fiz., No. 1 (1985).
4. V. I. Shatrov, "Method of calculation of MHD flow without separation around a sphere of incompressible liquid at Reynolds numbers $Re \leq 10^5$," ITPM Preprint No. 24-84, Institute of Theoretical and Applied Mechanics, Siberian Branch, Russian Academy of Sciences, Novosibirsk (1984).
5. V. I. Shatrov and V. I. Yakovlev, "The possibility of reducing the hydrodynamic drag during MHD flow around a sphere," Magn. Gidrodin., No. 1 (1990).
6. V. I. Shatrov and V. I. Yakovlev, "Optimization of the internal source in the problem of MHD flow around a self-propelled sphere," ITPM Preprint No. 19-89, Institute of Theoretical and Applied Mechanics, Siberian Branch, Russian Academy of Sciences, Novosibirsk (1989).
7. V. I. Khonchev and V. I. Yakovlev, "Contribution to the theory of a conduction-type MHD propeller with a free field," Prikl. Mekh. Tekh. Fiz., No. 5 (1980).